The group velocity

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Calculation of the group velocity and group refractive index for a waveguide mode.

I. SUMMARY

The group index n_g is calculated from effective refractive index n_{eff} as:

$$n_g = n_{eff} - \lambda \frac{\partial n_{eff}}{\partial \lambda} \tag{1}$$

where $v_p = \frac{c}{n_{eff}}$ is the phase velocity and $v_g = \frac{c}{n_g}$ is the group velocity

II. PROOF

The propagation constant of the waveguide mode is

$$\beta = \frac{\omega}{(c/n_{eff})} \tag{2}$$

The group velocity is

$$v_g = \frac{\partial \omega}{\partial \beta} == \left(\frac{\partial \beta}{\partial \omega} \right)^{-1} \tag{3}$$

Differentiating Eq. 2 dives

$$\frac{\partial \beta}{\partial \omega} = \frac{1}{c} \left(n_{eff} + \omega \frac{\partial n_{eff}}{\partial \omega} \right) \tag{4}$$

From Eq. 3

$$n_g = \frac{c}{v_g} = c \cdot \frac{\partial \beta}{\partial \omega} = n_{eff} + \omega \frac{\partial n_{eff}}{\partial \omega} \tag{5}$$

Since

$$\omega = \frac{2\pi}{\lambda} \tag{6}$$

The group index n_g is calculated as:

$$n_g = n_{eff} - \lambda \frac{\partial n_{eff}}{\partial \lambda} \tag{7}$$